

Noise Parameters Estimation with Gibbs Sampling for Localisation of Mobile Nodes in Wireless Networks

Lyudmila Mihaylova

Department of Communication Systems
Lancaster University, United Kingdom
mila.mihaylova@lancaster.ac.uk

Donka Angelova

Bulgarian Academy of Sciences, Central
Laboratory for Parallel Information Processing
Sofia, Bulgaria, donka@bas.bg

Abstract – *This paper presents a solution to the problem of self-localisation of mobile nodes in wireless sensor networks with unknown measurement noise characteristics. A Gibbs sampling algorithm estimates the unknown noise parameters followed by localisation of mobile nodes with a multiple model auxiliary particle filter (MM AUX-PF). The performance of the Gibbs sampler and MM AUX-PF is investigated in terms of accuracy and computational complexity using simulated and real received signal strength measurements. We show that the method provides accurate estimation results with complexity suitable for real-time applications.*

Keywords: Gibbs sampling, auxiliary particle filtering, localisation, wireless networks

1 Introduction

There is a great deal of methods for self-localisation of mobile nodes (e.g., [1–3]) between which the *range-based* [2] methods are widely used. They rely on the distances between nodes, usually evaluated using received signal strengths, and they vary in their complexity and accuracy. The range-based techniques can be divided into *radio frequency* (RF) ranging and *acoustic ranging*. The radio frequency ranging relies on the premise that by measuring the received signal strength a receiver can determine its distance to a transmitter.

While range-based algorithms need point-to-point distance estimation or angle estimation for positioning, *range-free* [4] algorithms do not require this information. Another classification subdivides the approaches to mobile nodes localisation in wireless networks to *indoor* and *outdoor*. In indoor sensor networks the problem can be solved with acoustic beacons (fixed or moving) or it is beacon free. Using moving beacons in mobile nodes positioning can significantly reduce the power consumption and costs.

From the point of view of the methods employed, a number of localisation techniques rely on Extended Kalman filters [5, 6], Monte Carlo techniques [7, 8], including nonparametric belief propagation [9], and on

the knowledge of the connectivity between nodes. Communications between nodes during the self-localisation are reduced to minimum due to energy and bandwidth constraints. In [8] multiple model particle filtering techniques for mobility tracking of users in cellular networks are developed. A particle filter, a Rao-Blackwellised particle filter are presented and their performance is compared with an Extended Kalman filter over simulated and real data from base stations. In [10, 11] an auxiliary MM PF is proposed for bearings-only tracking problems. In [3] an auxiliary PF is designed for target tracking in binary sensor networks.

None of the works mentioned above considers the case with unknown noise characteristics, nor the correlation in the slow shadowing component. However, in practice, the noise characteristics of the received signal strengths can vary in a large range, e.g., between 3dB and 24dB depending on the environment: urban or semi-urban, different meteorological conditions (snow, rain), presence of obscuration or attenuation of the signals and are typically correlated [12].

In this work, in contrast with existing papers in the literature, we propose a Gibbs sampling algorithm for estimating the unknown measurement noise parameters. The localisation of mobile nodes is performed after that with these noise parameter estimates embedded in a multiple model auxiliary particle filter. The proposed approach deals successfully with the highly nonlinear measurement models with non-Gaussian measurement errors, can incorporate physical constraints and possibly communications among frequently manoeuvring mobile nodes in the form of additional measurements. The performance of the proposed approach is investigated and analysed in terms of accuracy and complexity. In the considered formulation of the problem, node mobility is modeled as a linear system driven by a discrete-time command Markov process whereas the measurement models are nonlinear and necessitate a reliable nonlinear estimation method. Due to the fact that the control process of the nodes is unknown, node

mobility is modeled with multiple acceleration modes.

The following Section 2 formulates the considered problem for self-localisation of mobile nodes with unknown measurement noise characteristics and Section 3 presents a Gibbs sampler for noise measurement parameter estimation. A multiple model auxiliary particle filter for mobile nodes self-localisation is developed in Section 4. The performance of both filters is investigated in Section 5. Finally, Section 6 discusses the results and outlines open issues for future research.

2 Localisation of Mobile Nodes

Consider the two-dimensional problem of simultaneous localisation of n mobile nodes [2] where the positions $\{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}$ of the mobile nodes is estimated given n_r reference nodes with known coordinates $\{(x_{n+1}, y_{n+1}), (x_{n+2}, y_{n+2}), \dots, (x_{n+n_r}, y_{n+n_r})\}$ and pairwise measurements $\{z_{ij}\}$, where z_{ij} is a measurement between devices i and j . The reference nodes can obtain their coordinates in the network (through some external means such as GPS). Apart from their positions, the mobile nodes estimate their speeds and accelerations. This includes applications in which each sensor is equipped with a wireless transceiver and the distance between sensor locations is estimated by received signal strength indicator measurements or time delay of arrivals between sensor locations.

2.1 The Motion Model of Mobile Nodes

The state of each mobile node at time instant k is defined by the vector $\mathbf{x}_k = (x_k, \dot{x}_k, \ddot{x}_k, y_k, \dot{y}_k, \ddot{y}_k)'$ where x_k and y_k specify the position, \dot{x}_k and \dot{y}_k specify the speed, \ddot{x}_k and \ddot{y}_k are, resp., the acceleration in x and y directions in the two-dimensional plane; $'$ denotes the transpose operation. The motion of each mobile node can be described by the following Singer model [13]

$$\mathbf{x}_k = \mathbf{A}(T, \alpha)\mathbf{x}_{k-1} + \mathbf{B}_u(T)\mathbf{u}_k + \mathbf{B}_w(T)\mathbf{w}_k, \quad (1)$$

where $\mathbf{u}_k = (u_{x,k}, u_{y,k})'$ is a discrete-time command process. The respective matrices from (1) are in the form

$$\mathbf{A}(T, \alpha) = \begin{pmatrix} \tilde{\mathbf{A}} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \tilde{\mathbf{A}} \end{pmatrix}, \mathbf{B}_i(T) = \begin{pmatrix} \tilde{\mathbf{B}}_i & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 1} & \tilde{\mathbf{B}}_i \end{pmatrix}, \quad (2)$$

$$\tilde{\mathbf{A}} = \begin{pmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & \alpha \end{pmatrix}, \tilde{\mathbf{B}}_u = \begin{pmatrix} T^2/2 \\ T \\ 0 \end{pmatrix}, \tilde{\mathbf{B}}_w = \begin{pmatrix} T^2/2 \\ T \\ 1 \end{pmatrix}. \quad (3)$$

The subscript i in the matrix $\mathbf{B}(T)$ in (2) stands for u or w , respectively. The random process \mathbf{w}_k is a 2×1 vector and T is the discretisation period. The parameter α is the reciprocal of the manoeuvre time constant and thus depends on how long the manoeuvre lasts. The process noise \mathbf{w}_k is a white sequence, with covariance matrix $E[\mathbf{w}\mathbf{w}'] = \mathbf{Q} = \sigma_w^2 \mathbf{I}$, where $E[\cdot]$ is the mathematical expectation operation, \mathbf{I} denotes the unit matrix and σ_w is the standard deviation.

The unknown command processes $u_{x,k}$ and $u_{y,k}$ take values from a set of acceleration levels \mathcal{M}_x and \mathcal{M}_y . The process \mathbf{u}_k takes values from the set $\mathbb{M} = \mathcal{M}_x \times \mathcal{M}_y \triangleq \{\mathbf{u}_1, \dots, \mathbf{u}_r\}$. Let $\mathbb{S} \triangleq \{1, 2, \dots, r\}$ denote the set of models and let $m_k \in \mathbb{S}$ be the regime variable, modelled as a first-order Markov chain with transition probabilities $\Pi_{ij} = P(m_k = j | m_{k-1} = i)$, $i, j = 1, \dots, r$ and initial modes probability $\tilde{\mu}_{i,0} = P_0\{m = m_i\}$ for $m_i \in \mathbb{S}$ such that $\tilde{\mu}_{i,0} \geq 0$ and $\sum_{i=1}^r \tilde{\mu}_{i,0} = 1$.

2.2 Observation Model

In wireless networks, the distance between mobile and reference nodes can be inferred from received signal strength indicators (RSSIs) or pilot signals of sensor nodes. The propagation of RSSIs in different types of environment (dense urban, semi-urban or desert) can be described by the model [1, 14]

$$z_{\ell j,k} = \kappa_\ell - 10\gamma \log_{10}(d_{\ell j,k}) + v_{\ell j,k}. \quad (4)$$

The RSSI $z_{\ell j,k}$ is received at the mobile node N_ℓ with coordinates $(x_{\ell,k}, y_{\ell,k})$ at time k , after being transmitted from the node N_j with coordinates $(x_{j,k}, y_{j,k})$; κ_ℓ is a constant depending on the transmission power, wavelength, and gain of node N_ℓ , γ is the slope index (typically $\gamma = 2$ for highways and $\gamma = 4$ for microcells in urban environment), $v_{\ell j,k}$ is the measurement noise assumed to be Gaussian, with unknown mean μ_v and unknown standard deviation σ_v , i.e., $v_{\ell j,k} \sim \mathcal{N}(\mu_v, \sigma_v^2)$, and $d_{\ell j,k}$ is the distance between mobile nodes N_ℓ and N_j $d_{\ell j,k} = \sqrt{(x_{\ell,k} - x_{j,k})^2 + (y_{\ell,k} - y_{j,k})^2}$. All mobile nodes in the group send their pilot signals to some reference nodes. In order to locate a single mobile node in the two-dimensional plane, the three largest RSSIs to reference neighbouring nodes are necessary to enable triangulation. In the general case with $\ell = 1, \dots, n$ mobile nodes and $j = 1, \dots, r$ reference nodes, when there are communications between the mobile nodes and reference nodes, the overall observation vector $\mathbf{z}_k \in \mathbb{R}^{n_r}$ will contain $L = n \cdot r$ number of measurements. A more general form of (4) is

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k, \quad (5)$$

where $\mathbf{h}(\mathbf{x}_k)$ is a nonlinear function with components $h_{\ell j,k} = \kappa_\ell - 10\gamma \log_{10}(d_{\ell j,k})$, $j = 1, 2, 3$. The noise \mathbf{v}_k characterises the shadowing components, assumed to be uncorrelated in time and with unknown parameters. One simple, but effective solution to the localisation problem with unknown noise parameters is to model the measurement noise as a n_{mix} -component Gaussian mixture

$$v_k \sim \sum_{i=1}^{n_{mix}} \pi_i \mathcal{N}(\mu_i, \sigma_i^2) \quad (6)$$

where μ_i and σ_i^2 are the mean and variance of the mixture component i and $\boldsymbol{\pi} = (\pi_1, \dots, \pi_{n_{mix}})$ is a vector of mixture weights π_i (constrained to be non-negative

and with unit sum). The features of the measurement process and environment, the availability of missed or false observations can be captured well by the mixture.

The Bayesian approach is very powerful for approximate estimation of the mixture parameters, by introducing a hierarchical data structure of the mixture model and accounting for the missing data. In particular, Markov chain Monte Carlo (MCMC) techniques are very efficient inference methods. For hierarchical models, Gibbs sampling has proved to be the most effective among various MCMC methods. Gibbs sampling is especially appropriate for the localisation problem under unknown measurement parameters. The mixture can be composed of different distributions such as t , Student or Gaussian. We choose Gaussian components since they lead easily to tractable solutions and can model well complex probability density functions even with a small number of components [15].

We estimate the measurement noise parameters with the Gibbs sampler presented in Section 3 and next the mobile nodes self-localisation is performed by the Multiple Model Auxiliary Particle Filter given in Section 4.

2.3 General Models for Simultaneous Localisation of Mobile Nodes

A combined state vector $\mathbf{X}_k = \{\mathbf{x}'_{1,k}, \dots, \mathbf{x}'_{n,k}\}$ is composed and all states of the mobile nodes are simultaneously estimated. The motion model (1)-(3) can be generalised to the form

$$\mathbf{X}_k = \mathbf{f}(\mathbf{X}_{k-1}, \mathbf{M}_k, \mathbf{U}_k, \mathbf{W}_k), \quad (7)$$

where $\mathbf{X}_k \in \mathbb{R}^{n \times n_x}$ is the combined system *base* state vector, $\mathbf{U}_k \in \mathbb{R}^{n \times n_u}$ specifies the command processes for all mobile nodes, and the *modal (discrete) state* $\mathbf{M}_k \in \mathbb{S}$ of the different system modes (regimes); $\mathbf{W}_k \in \mathbb{R}^{n \times n_w}$ is the combined vector of the system noise.

A generalised form of the measurement equation (5) is

$$\mathbf{Z}_k = \mathbf{h}(\mathbf{X}_k) + \mathbf{V}_k, \quad (8)$$

where $\mathbf{Z}_k \in \mathbb{R}^{n \times n_z}$ is a generalised measurement vector, and the generalised noise vector $\mathbf{V}_k \in \mathbb{R}^{n \times n_z}$ characterises the shadowing components.

3 Gibbs Sampling for Noise Parameter Estimation

We suggest to estimate the unknown noise parameters of the measurement model (4)-(5) with Gibbs sampling (GS), a special form of Bayesian sampling benefiting from the hierarchical structure of the model [16]. Given the observation of a T_m -dimensional vector $\boldsymbol{\eta} = (\eta_1, \dots, \eta_{T_m}) \in \mathbb{R}^{T_m}$ of independent random variables (corresponding to $\mathbf{v} = (v_{\ell j,1}, v_{\ell j,2}, \dots, v_{\ell j, T_m})^1$), the mixture [17] is formed

¹From the measurement model (4), estimated positions (x_ℓ, y_ℓ) , estimated distance $d_{\ell,j}$ and with known $\kappa_{\ell,j}$, noise realisations $v_{\ell j,1:T_m}$ are found which serve as measurements to GS.

$$F(\boldsymbol{\eta}_t) = \sum_{i=1}^{n_{mix}} \pi_i F_i(\boldsymbol{\eta}_t), \quad t = 1, \dots, T_m, \quad (9)$$

where the densities F_i , $i = 1, \dots, n_{mix}$ are known or are known up to a parameter. The weight vector $\boldsymbol{\pi} = (\pi_1, \dots, \pi_{n_{mix}})$ has non-negative components π_i which sum is equal to 1. We are representing the measurement noise by a Gaussian mixture model (GMM): the density $F_i(\boldsymbol{\eta}_t)$ is then Gaussian, $\mathcal{N}(\boldsymbol{\mu}_i, \boldsymbol{\sigma}_i^2)$, where $\boldsymbol{\mu}_i$ and $\boldsymbol{\sigma}_i^2$ are the mean and variance of the i -th mixture component. The unknown parameters and weights of the GMM, denoted by $\boldsymbol{\theta} = (\boldsymbol{\theta}_1, \dots, \boldsymbol{\theta}_{r_m}) = (\boldsymbol{\mu}_1, \boldsymbol{\sigma}_1^2, \dots, \boldsymbol{\mu}_{n_{mix}}, \boldsymbol{\sigma}_{n_{mix}}^2, \pi_1, \dots, \pi_{n_{mix}})$ are iteratively estimated.

According to [17], the mixture model can be represented in terms of missing (or incomplete) data. The model is *hierarchical* with the true parameter vector $\boldsymbol{\theta}$ of the mixture, on the top level and on the bottom are the observed data. The GS relies additionally on the availability of all complete conditional distributions of the elements of $\boldsymbol{\theta}$, breaking down $\boldsymbol{\theta}$ into r_m subsets. It generates $\boldsymbol{\theta}_j, j = 1, \dots, r_m$ conditional on all the other parameters, increasing in this way the number of conditional simulations. The details of the GS algorithm can be found in a number of publications [17, 18]. The n_{mix} -dimensional vectors $\boldsymbol{\delta}_t, t = 1, 2, \dots, T_m$ with components $\delta_{t,i} \in \{0, 1\}, i = 1, 2, \dots, n_{mix}$ and $\sum_{i=1}^{n_{mix}} \delta_{t,i} = 1$ are defined to indicate that the measurement $\boldsymbol{\eta}_t$ has density $F_i(\boldsymbol{\eta}_t)$. Next, the missing data distribution depends on $\boldsymbol{\theta}, \boldsymbol{\delta} \sim p(\boldsymbol{\delta}|\boldsymbol{\theta})$. The observed data, $\boldsymbol{\eta} \sim p(\boldsymbol{\eta}|\boldsymbol{\theta}, \boldsymbol{\delta})$, are at the bottom level. Bayesian sampling iteratively generates parameter vectors $\boldsymbol{\theta}^{(u)}$ and missing data $\boldsymbol{\delta}^{(u)}$ according to $p(\boldsymbol{\theta}|\boldsymbol{\eta}, \boldsymbol{\delta}^{(u)})$ and $p(\boldsymbol{\delta}|\boldsymbol{\eta}, \boldsymbol{\theta}^{(u+1)})$. Here, u indexes the current iteration.

It is proven in [17], that Bayesian sampling produces an ergodic Markov chain $(\boldsymbol{\theta}^{(u)})$ with stationary distribution $p(\boldsymbol{\theta}|\boldsymbol{\eta})$. After u_0 initial (warming up) steps, a set of U samples $\boldsymbol{\theta}^{(u_0+1)}, \dots, \boldsymbol{\theta}^{(u_0+U)}$ are approximately distributed as $p(\boldsymbol{\theta}|\boldsymbol{\eta})$. Due to ergodicity, averaging can be made with respect to time and the empirical mean of the last U values can be used as an estimate of $\boldsymbol{\theta}$.

The choice of prior distributions and their hyperparameters is a first, important step of the design of a Gibbs sampler. Conjugate prior distributions are chosen (as in most cases in the literature) since this simplifies the implementation. Since the conjugate prior of $\boldsymbol{\pi}$ is a *Dirichlet* distribution (DD), $\mathcal{D}(\alpha_1, \dots, \alpha_{n_{mix}})$ [17], $\boldsymbol{\pi}$ is generated according to the DDs with parameters, depending on the missing data. The conjugate priors for $\boldsymbol{\sigma}_i^2$ and $\boldsymbol{\mu}_i|\boldsymbol{\sigma}_i^2, i = 1, \dots, n_{mix}$ are the *Inverse Gamma* (\mathcal{IG}) and *Gaussian* distribution respectively, as recommended in [17]:

$$\boldsymbol{\pi} \sim \mathcal{D}(\alpha_1, \dots, \alpha_{n_{mix}}), \quad (10)$$

$$\boldsymbol{\sigma}_i^2 \sim \mathcal{IG}(\nu_i, s_i^2), \quad (11)$$

$$\boldsymbol{\mu}_i|\boldsymbol{\sigma}_i^2 \sim \mathcal{N}(\xi_i, \boldsymbol{\sigma}_i^2/n_i). \quad (12)$$

Here, $\alpha_i, \nu_i, s_i^2, \xi_i$, and $n_i, i \in 1, \dots, n_{mix}$ are the corresponding hyperparameters.

Starting with an initial parameter vector $\boldsymbol{\theta}^{(0)}$, the following *iterative algorithm* is implemented: at the iteration $u, u = 0, 1, 2, \dots$

a) generate $\boldsymbol{\delta}^{(u)} \sim p(\boldsymbol{\delta}|\boldsymbol{\eta}, \boldsymbol{\theta}^{(u)})$ from a multinomial distribution with weights proportional to the observation likelihoods, i.e. $p(\delta_{ti}^{(u)}) \propto \pi_i^{(u)} \mathcal{N}(\eta_t; \mu_i^{(u)}, \sigma_i^{(u)^2})$;

b) generate $\boldsymbol{\pi}^{(u+1)} \sim p(\boldsymbol{\pi}|\boldsymbol{\eta}, \boldsymbol{\delta}^{(u)})$:

$$\boldsymbol{\pi}^{(u+1)} \sim \mathcal{D}\left(\alpha_1 + \Delta_1^{(u)}, \dots, \alpha_{n_{mix}} + \Delta_{n_{mix}}^{(u)}\right)$$

where $\Delta_i^{(u)} = \sum_{t=1}^{T_m} \delta_{t,i}^{(u)}$ is the number of observations allocated to the mixture component i ;

c) generate $\mu_i^{(u+1)}|\sigma_i^{(u)^2} \sim \mathcal{N}\left(\xi_i, \sigma_i^{(u)^2}/n_i\right)$:

$$\mu_i^{(u+1)} \sim \mathcal{N}\left(\frac{n_i \xi_i + \sum_{t=1}^{T_m} \delta_{t,i}^{(u)} \eta_t}{n_i + \Delta_i^{(u)}}, \frac{\sigma_i^{(u)^2}}{n_i + \Delta_i^{(u)}}\right)$$

where ξ_i is the average of the observations attributed to the mixture component i ;

d) generate

$$\sigma_i^{(u+1)^2} \sim \mathcal{IG}\left(\nu_i + \nu_i, s_i^2 + \hat{s}_i^2 + n_i(\xi_i - \mu_i^{(u+1)})^2\right)$$

where $\nu_i = 0.5(\Delta_i^{(u)} + 1)$, $s_i^2 = (T_m(n_{mix} - 1))^{-1} \sum_{t=1}^{T_m} (\eta_t - \bar{\eta})^2$, ($\bar{\eta}$ is the empirical mean of the measurement data), $\hat{s}_i^2 = \sum_{t=1}^{T_m} \delta_{t,i}^{(u)} (\eta_t - \mu_i^{(u+1)})^2$; We consider the additional assumption that $\sigma_i^2 \in [\sigma_{min}^2, \sigma_{max}^2]$. Truncated conjugate priors are still conjugate [17]. Sampling is realised by generating $\sigma_i^{(u)^2}$ from the \mathcal{IG} distribution until $\sigma_i^{(u)^2} \in [\sigma_{min}^2, \sigma_{max}^2]$.

e) calculate the output estimate $\hat{\boldsymbol{\theta}} = \frac{1}{U} \sum_{l=1}^U \boldsymbol{\theta}^{(u_0+l)}$.

In the present implementation, the observation η_t coincides with the RSSI measurement error.

4 A Multiple Model Auxiliary Particle Filter for Localisation

4.1 The Particle Filtering Framework

Within particle filtering the localisation of mobile nodes reduces to approximation of the state probability density function (PDF) given a sequence of measurements. According to the Bayes' rule the filtering PDF $p(\mathbf{X}_k|\mathbf{Z}_{1:k})$ of the state vector $\mathbf{X}_k \in \mathbb{R}^{n \times n_x}$ given a sequence of sensor measurements $\mathbf{Z}_{1:k}$ up to time k can be written as

$$p(\mathbf{X}_k|\mathbf{Z}_{1:k}) = \frac{p(\mathbf{Z}_k|\mathbf{X}_k)p(\mathbf{X}_k|\mathbf{Z}_{1:k-1})}{p(\mathbf{Z}_k|\mathbf{Z}_{1:k-1})}, \quad (13)$$

where $p(\mathbf{Z}_k|\mathbf{Z}_{1:k-1})$ is the normalising constant. The state *predictive* distribution is given by the Chapman-

Kolmogorov equation

$$p(\mathbf{X}_k|\mathbf{Z}_{1:k-1}) = \int_{\mathbb{R}^{n \times n_x}} p(\mathbf{X}_k|\mathbf{X}_{k-1})p(\mathbf{X}_{k-1}|\mathbf{Z}_{1:k-1})d\mathbf{X}_{k-1}. \quad (14)$$

The evaluation of the right hand side of (13) involves integration which can be avoided in the particle filtering approach [19] by approximating the posterior PDF $p(\mathbf{X}_k|\mathbf{Z}_{1:k})$ with a set of particles $\mathbf{X}_{0:k}^{(i)}$, $i = 1, \dots, N$ and their corresponding weights $w_k^{(i)}$. Then the posterior density can be written as follows

$$p(\mathbf{X}_{0:k}|\mathbf{Z}_{1:k}) = \sum_{i=1}^N w_k^{(i)} \delta(\mathbf{X}_{0:k} - \mathbf{X}_{0:k}^{(i)}), \quad (15)$$

where $\delta(\cdot)$ is the Dirac delta function, and the weights are normalised such that $\sum_i w^{(i)} = 1$.

Each pair $\{\mathbf{X}_{0:k}^{(i)}, w_k^{(i)}\}$ characterises the belief that the object is in state $\mathbf{X}_{0:k}^{(i)}$. An estimate of the variable of interest is obtained by the weighted sum of particles. Two major stages can be distinguished: *prediction* and *update*. During prediction, each particle is modified according to the state model, including the addition of random noise in order to simulate the effect of the noise on the state. In the update stage, each particle's weight is re-evaluated based on the new data. The *resampling* [20] procedure is applied here to introduces variety in the particles by eliminating those with small weights and replicating the particles with larger weights such that the approximation in (15) still holds.

Since the command process of the mobile nodes is unknown, a multiple model auxiliary particle filter (MM AUX-PF) is designed for localisation of the mobile nodes. Given the set \mathbb{M} covering well the possible command values, the unknown commands are supposed to evolve as a first-order Markov chain with transition probability matrix $\boldsymbol{\Pi}$. The particles for the base state are generated from the transition prior, according to (7)-(8) (with motion model (1)-(3) for each mobile).

4.2 Auxiliary Multiple Model Particle Filtering for Localisation

The auxiliary Sampling Importance Resampling (SIR) PF was introduced by Pitt and Shephard [21]. The auxiliary PF draws particles from an importance function which is close as possible to the optimal one. The auxiliary PF introduces an importance function $q(\mathbf{X}_k, i^{(j)})_{i=1}^N$ where $i^{(j)}$ refers to the index of the particle at $k-1$. The filter obtains samples from the joint density $p(\mathbf{X}_k, i|\mathbf{Z}_{1:k})$ and then omits the index i in the pair (\mathbf{X}_k, i) to produce a sample $\{\mathbf{X}_k^{(i)}\}_{i=1}^N$ from the marginalised density $p(\mathbf{X}_k, |\mathbf{Z}_{1:k})$. The importance density that generates the sample $\{\mathbf{X}_k^{(i)}\}_{i=1}^N$ is defined to satisfy the relation [19]

$$q(\mathbf{X}_k, i|\mathbf{Z}_{1:k}) \propto p(\mathbf{Z}_k|\boldsymbol{\mu}_k^{(i)})p(\mathbf{X}_k|\mathbf{X}_{k-1}^{(i)})w_{k-1}^{(i)}, \quad (16)$$

where $\boldsymbol{\mu}_k^{(i)}$ is some characteristics of \mathbf{X}_k given $\mathbf{X}_{k-1}^{(i)}$.

The selection of the most promising particles is carried out by sampling from a multinomial distribution where the number of possible outcomes is N^{out} . The auxiliary PF [21] resamples the predicted particles to select which particles to use in the prediction and measurement update.

For the purposes of mobile node localisation we propose a MM AUX-PF. The MM AUX-PF represents the PDF $p(\mathbf{X}_k, i, \mathbf{M}_k | \mathbf{Z}_{1:k})$ where i refers to the i -th particle at $k-1$. After marginalisation, the representation of $p(\mathbf{X}_k | \mathbf{Z}_{1:k})$ can be obtained.

Then the importance sampling function (16) can be re-written in the form

$$q(\mathbf{X}_k, i | \mathbf{Z}_{1:k}) = q(i | \mathbf{Z}_{1:k})q(\mathbf{X}_k | i, \mathbf{Z}_{1:k}) \quad (17)$$

and defining $q(\mathbf{X}_k, i | \mathbf{Z}_{1:k}) \triangleq p(\mathbf{X}_k | \mathbf{X}_{k-1}^{(i)})$ it follows from (16) that

$$q(i | \mathbf{Z}_{1:k}) \propto p(\mathbf{Z}_k | \boldsymbol{\mu}_k^{(i)})w_{k-1}^{(i)}. \quad (18)$$

Hence, the weights are calculated according to

$$w_k^{(i)} \propto p(\mathbf{Z}_k | \boldsymbol{\mu}_k^{(i)}). \quad (19)$$

Selection of the Most Promising Particles

For the selection of the most promising particles, the conditional mean $\boldsymbol{\mu}_k^{(i)}$

$$\boldsymbol{\mu}_k^{(i)} = E(\mathbf{X}_k | \mathbf{X}_{k-1}^{(i)}) \quad (20)$$

of $\boldsymbol{\mu}_k^{(i)}$ given $\mathbf{X}_{k-1}^{(i)}$ is used. The conditional mean $\boldsymbol{\mu}_k^{(i)}$ for each particle in the MM AUX-PF comprises the mean vectors of all mobile nodes and can be calculated from:

$$\mathbf{x}_k = \mathbf{A}(T, \alpha)\mathbf{x}_{k-1} + \mathbf{B}_u(T)\mathbf{u}_k. \quad (21)$$

The MM AUX-PF for mobile nodes localisation is presented as Algorithm 2. It takes into account speed constraint V_{max} , for each mobile node. Finally, resampling is performed only when the efficient number of particles, N_{eff} is smaller than a given threshold N_{thresh} .

5 Performance Evaluation

5.1 Results with Simulated Data

Let us suppose, that the trajectory of the ℓ -th mobile node is provided by a GPS system, which collects its actual positions during a time interval $t = 1, \dots, T_m$ with sampling period T . Knowing the distance to the j -th reference node and using the RSSI measurements $z_{\ell j, t}$, the measurement error parameters can be estimated. A sample of measurement errors $v_{\ell j, t}$, $t = 1, \dots, T_m$ can be obtained also, if the mobile mode is static for some time interval or if it is moving along a road with known parameters (the route map is available). In the univariate case, where all mobile nodes have the same noise

Algorithm 2. A MM AUX-PF for Localisation

Initialisation

I. $k = 0$, for $i = 1, \dots, N$,
Generate samples $\{\mathbf{X}_0^{(i)} \sim p(\mathbf{X}_0), \mathbf{M}_0^{(i)} \sim P_0(\mathbf{M})\}$
and set initial weights $w_0^{(i)} = 1/N$.

II. For $k = 1, 2, \dots$,

(1) For $i = 1, \dots, N * r$,

- Calculate the conditional mean:

$$\boldsymbol{\mu}_k^{(i)}(\mathbf{M}_k) = E(\mathbf{X}_k | \mathbf{X}_{k-1}^{(i)}, \mathbf{M}_k) \text{ for every } \mathbf{M}_k \in \mathbb{S}$$

(2) Generate $\{i^j, \mathbf{M}_k^{(j)}\}$, $j = 1, \dots, N$ by sampling from $q(i, \mathbf{M}_k | \mathbf{Z}_{1:k})$, where

$$q(i, \mathbf{M}_k | \mathbf{Z}_{1:k}) \propto p(\mathbf{Z}_k | \boldsymbol{\mu}_k^{(i)}(\mathbf{M}_k))p(\mathbf{M}_k | \mathbf{M}_{k-1}^{(i)})w_{k-1}^{(i)}.$$

(3) **Prediction Step**

For $j = 1, \dots, N$, predict the particles according to

$$\mathbf{X}_k^{(j)} = f(\mathbf{X}_{k-1}^{(i^j)}, \mathbf{M}_k^{(j)}, \mathbf{w}_k^{(j)}), \mathbf{w}_k^{(j)} \sim \mathcal{N}(0, \mathbf{Q}).$$

Account for the speed constraints.

(4) **Measurement update**

For $j = 1, \dots, N$ compute the weights

$$w_k^{(j)} = p(\mathbf{Z}_k | \mathbf{X}_k^{(j)})/p(\mathbf{Z}_k | \boldsymbol{\mu}_k^{(i^j)}(\mathbf{M}_k)).$$

Normalise the weights: $\tilde{w}_k^{(j)} = w_k^{(j)} / \sum_{j=1}^N w_k^{(j)}$

(5) **Output estimate**

The posterior mean $E[\mathbf{X}_k | \mathbf{z}_k]$

$$\hat{\mathbf{x}}_k = \sum_{j=1}^N \tilde{w}_k^{(j)} \mathbf{x}_k^{(j)}.$$

(6) **Resampling step:**

Compute the effective sample size

$$N_{eff} = 1 / \sum_{j=1}^N (\tilde{w}_k^{(j)})^2,$$

Resample if $N_{eff} < N_{thresh}$

* For $i = 1, \dots, N$, set $w_k^{(i)} = 1/N$.

statistics $v_{\ell j, t} = v_t$, $t = 1, \dots, T_m$, the noise characteristics can be assessed preliminarily, improving in this way the filter performance. GS for estimating mixture parameters is investigated over simulated and real data.

We have selected the following hyperparameters for every $i = 1, \dots, n_{mix}$: $n_i = 1$, $\nu_i = n_{mix}$, if $\alpha_i = 1$, the Dirichlet distribution reduces to a uniform distribution and the algorithm is initialised with noninformative prior about mixture proportions. The initial values of $\boldsymbol{\theta}^{(0)}$ are chosen based on the prior information about physical restrictions on the parameters: $\sigma_i^{(0)} = 6$ [dB], the bounds of the supplementary assumption $\sigma_i^2 \in [\sigma_{min}^2, \sigma_{max}^2]$ are respectively $[1^2, 20^2]$. Initial mean values $\mu_i^{(0)}$ are calculated based on the observed interval of variation of the data. The initial weights are selected $\pi_i^{(0)} = 1/n_{mix}$. The number of iterations is 250 and the initial ‘warming up’ interval is $u_0 = 100$.

We performed experiments over the scenario shown in Figure 1 after evaluating the noise parameters first. A sample of $T_m = 2000$ measurement errors is generated

according to the following mixture model with $n_{mix} = 2$ elements: $v_t \sim 0.2\mathcal{N}(-6.5, 2^2) + 0.8\mathcal{N}(8.0, 4^2)$. The histogram of the modeled measurement errors is presented in Figure 2 (a). The two modes are well pronounced on this Figure. The estimated mixture parameters are $\hat{\pi} = (0.19, 0.81)$, $\hat{\mu}_1 = -6.9$, $\hat{\sigma}_1^2 = 4$, $\hat{\mu}_2 = 8.04$, $\hat{\sigma}_2^2 = 17$. The mixture PDF approximation of the noise is given in Figure 2 (b).

It is assumed that the accelerations of the mobile nodes u_x and u_y can change within the range $[-5, 5]$ [m/s^2] and that the command process \mathbf{u} takes values among the following acceleration levels $\mathbb{M} = \{(0, 0)', (3.5, 0)', (0, 3.5)', (0, -3.5)', (-3.5, 0)'\}$. Thus the number of motion modes is $M = 5$. Non-random mobile node trajectories were generated with the dynamic state equation (1)-(3) without process noise.

The reference nodes in the scenario from Figure 1 are randomly deployed on the observed area. The MM AUX-PF performance with noise statistics estimated by GS is compared with the filter performance with inaccurate noise distributions: in the first case the measurement noise statistics are assumed Gaussian with parameters: $v_t \sim \mathcal{N}(0, \sigma_{mix}^2)$, where $\sigma_{mix}^2 = 6^2$ is the mixture variance, and in the second case $v_t \sim \mathcal{N}(0, 4^2)$, parameters typical for urban environment. The respective position and speed RMSE are given in Figures 3 (a) and (b). The experiments show that accu-

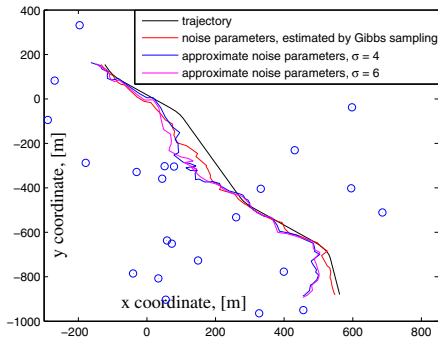


Figure 1: A mobile unit moving in an area covered with randomly deployed wireless sensors with unknown measurement noise characteristics. Estimated trajectories are shown obtained by the MM AUX-PF: 1: noise parameters estimated with the GS: $v_k \sim \sum_{i=1}^2 \hat{\pi}_i \mathcal{N}(\hat{\mu}_i, \hat{\sigma}_i^2)$, 2: $v_k \sim \mathcal{N}(0, 6^2)$, 3: $v_k \sim \mathcal{N}(0, 4^2)$

rate noise parameters are improving the localisation accuracy for deterministically deployed sensor networks. However, they are especially useful for randomly deployed networks. It can be seen from Figure 3 (a) and (b) that the estimation errors are larger where the manoeuvring phases of mobile nodes coincide with the places of sparsely located reference nodes. The peak dynamic errors increase when the information about noise parameters is insufficient. The position and speed root-mean-square errors (RMSEs) are minimum when the filter operates with estimated noise parameters. If

the mean (case 2 in Figure 3) or mean and variance (case 3) of the noise are unknown, the errors increase progressively. The parameters of the state vector

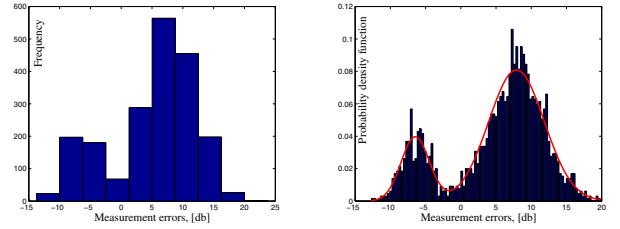


Figure 2: (a) Histogram of the modeled noise, (b) Noise histogram overlaid with mixture density estimate

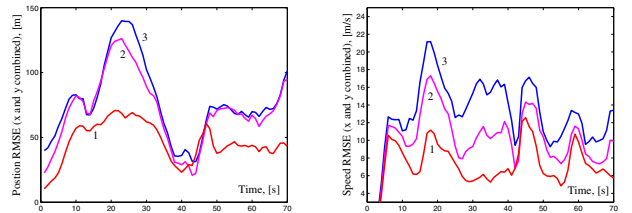


Figure 3: Position RMSE combined (for x and y) and speed RMSEs (for \dot{x} and \dot{y}): 1: $v_k \sim \sum_{i=1}^2 \hat{\pi}_i \mathcal{N}(\hat{\mu}_i, \hat{\sigma}_i^2)$, 2: $v_k \sim \mathcal{N}(0, 6^2)$, 3: $v_k \sim \mathcal{N}(0, 4^2)$

initial distribution $\mathbf{x}_{1,0} \sim \mathcal{N}(\mathbf{m}_{1,0}, \mathbf{P}_{1,0})$ are selected as follows: $\mathbf{P}_{1,0} = \text{diag}\{\mathbf{P}_{x,0}, \mathbf{P}_{y,0}\}$, $\mathbf{P}_{x,0} = \mathbf{P}_{y,0} = \text{diag}\{30 [\text{m}]^2, 1 [\text{m/s}]^2, 0.5 [\text{m/s}^2]^2\}$, and $\mathbf{m}_{1,0}$ contains the exact initial node states. Initial mode probabilities are $\tilde{\mu}_{1,0} = 0.8$ and $\tilde{\mu}_{i,0} = 0.05$ for $i = 2, \dots, 5$. The transition probability matrix $\mathbf{\Pi}$ has diagonal elements: $\Pi_{11} = 0.9, \Pi_{ii} = 0.7, i = 2, \dots, 5$. The other parameters of the algorithm are given in Table 1.

The RMSE combined on both position coordinates are used to assess the closeness of the estimated state parameters to the actual dynamic parameters of mobile nodes over $N_{mc} = 50$ Monte Carlo runs.

Table 1. Simulation parameters

Discretisation time step T	1.0 [s]
Correlation coefficient a	0.35
Path loss index γ	4
Transmission power κ	30
Variance σ_w^2 of the noise \mathbf{w}_k in (1)	$0.5^2 [\text{m/s}^2]^2$
Maximum speed V_{\max}	45 [m/s]
Number of particles	$N = 500$
Threshold for resampling	$N_{\text{thresh}} = N/10$
Number of Monte Carlo runs	$N_{mc} = 50$
Variance σ_v^2 of the noise v_k	$4^2 [\text{dB}]^2$

5.2 Results with Real Data

The performance of the GS algorithm has been investigated with real RSSIs, collected from base stations (BSs) in Glasgow, United Kingdom. The mobile station (MS) was a vehicle driving in the city centre. More

than 400 BSs are available in the area where the car operated. A GPS system collected the actual positions of the moving MS. The data from three BSs are used to evaluate the noise characteristics. The sample size is $T_m = 800$. The noise histogram and density estimate, obtained by fitting the mixture parameters to the data are presented in Figure 4 (a), (b) and (c). The number of iterations and the “warming up” bound are selected to 550 and 450 respectively. The initial standard deviation is chosen as $\sigma^{(0)} = 1.5$ [dB], with an additional sampling restriction of $\sigma_{min}^2 = 0.001^2$ and $\sigma_{max}^2 = 10^2$. The actual trajectory of the mobile is shown on Figure 6 and the estimated trajectories are given on Figure 5 both without and with GS for estimating the noise parameters. The position RMSE is presented on Figure 7. For the GS algorithm

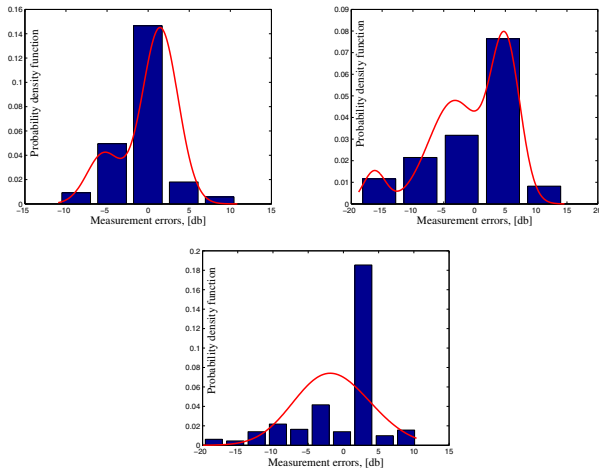


Figure 4: Noise histogram and density estimate with $n_{mix} = 2$ components for: (a) BS1, (b) BS2, (c) BS3

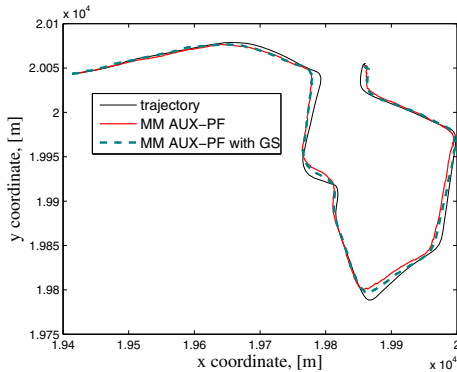


Figure 5: Actual and estimated trajectories obtained by the MM AUX-PF: 1: $v_k \sim \mathcal{N}(0, 6^2)$, 2. noise parameters estimated with the GS, for BS_1 , $v_k \sim \sum_{i=1}^2 \hat{\pi}_i \mathcal{N}(\hat{\mu}_i, \hat{\sigma}_i^2)$, for BS_2 $v_k \sim \sum_{i=1}^3 \hat{\pi}_i \mathcal{N}(\hat{\mu}_i, \hat{\sigma}_i^2)$ and for BS_3 , $v_k \sim \sum_{i=1}^2 \hat{\pi}_i \mathcal{N}(\hat{\mu}_i, \hat{\sigma}_i^2)$.

when the data histograms have clearly differentiated modes (such as in Figure 2), the number of mixture components can be easily determined and this facili-

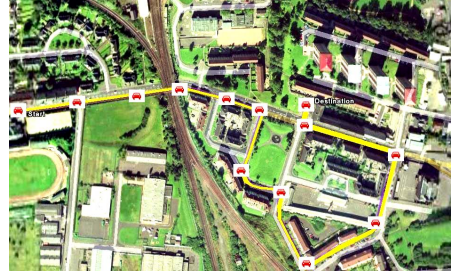


Figure 6: Actual trajectory of the mobile node

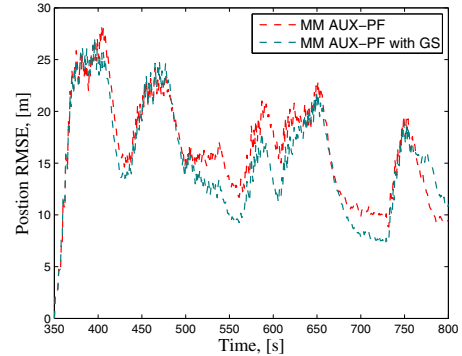


Figure 7: Position RMSE (combined for x and y) of the MM AUX-PF: 1: $v_k \sim \mathcal{N}(0, 6^2)$, 2. noise parameters estimated with the GS, for BS_1 , $v_k \sim \sum_{i=1}^2 \hat{\pi}_i \mathcal{N}(\hat{\mu}_i, \hat{\sigma}_i^2)$, for BS_2 $v_k \sim \sum_{i=1}^3 \hat{\pi}_i \mathcal{N}(\hat{\mu}_i, \hat{\sigma}_i^2)$ and for BS_3 , $v_k \sim \sum_{i=1}^2 \hat{\pi}_i \mathcal{N}(\hat{\mu}_i, \hat{\sigma}_i^2)$.

tates the mixture parameters evaluation. When the modes are difficult to distinguish (such as in Figure 4 (c)), a small number of mixture components can be used. However, this is not an obstacle in the present application. As a result from the GS, the following estimates are obtained for the noise parameters of the RSSI from the three base stations: BS_1 ($n_{mix} = 2$) and mixture estimates $\hat{\pi} = (0.2, 0.79)'$, $\hat{\mu} = (-5.4, 1.46)'$, $\sigma_{v,1} = 4.15$, $\sigma_{v,2} = 4.7$; BS_2 ($n_{mix} = 3$) with mixture estimates $\hat{\pi} = (0.07, 0.5, 0.4)'$, $\hat{\mu} = (-16.25, -3.26, 5.02)'$, $\sigma_{v,1} = 3.22$, $\sigma_{v,2} = 17.59$, $\sigma_{v,3} = 5.6$; BS_3 ($n_{mix} = 2$) with mixture estimates $\hat{\pi} = (0.48, 0.52)'$, $\sigma_{v,1} = 20.27$, $\sigma_{v,2} = 25.6$.

The computational complexity is another important issue that we investigated. The MM AUX-PF execution time increases with the number of maneuvering models used in the filter. The ratio between the computational time of the MM AUX-PF with 5 models and the computational time of the conventional AUX-PF is approximately 3:1. In MATLAB environment, one-step processing time of a mobile node is approximately 2 seconds on a conventional PC, with the MM AUX-PF. By using C++ programming tools the computational time is reduced to the sampling interval. The GS execution time in the case of 550 iterations and sample size $T_m = 800$ is approximately 30 sec. With a sample size $T_m = 2000$, the computational time is less than 2

minutes.

Since we have data only with RSSI for outdoor environment, we show results with real data only for outdoor localisation. We expect in the case of indoor localisation the effect of the Gibbs sampling and parameter estimation to be very pronounced.

6 Conclusions

This paper contributes to solving the problem of mobile nodes localisation in wireless networks with unknown measurement noise characteristics. A Gibbs sampler is proposed for noise parameter estimation. The estimation of the noise parameters is especially useful for randomly deployed networks and in areas where the measurement noise varies in a wide range. A MM auxiliary particle filter is proposed for localisation of mobile nodes in wireless sensor networks. The algorithms' performance has been investigated and validated over different scenarios. The combination of Gibbs sampling and MM AUX-PF gives high accuracy for localising manoeuvring nodes. The developed techniques can be applied to GPS-free position localisation of mobile nodes in wireless networks, including in scenarios where the location information for the mobile nodes is supporting basic network functions. Future work will be focused on localisation when both fixed and mobile nodes communicate with each other, on techniques for sensor clustering and connectivity and on prototype wireless networks of acoustic sensors.

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